Search- & Meta-heuristics

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1 Introduction

This report aims to provide insight into the first programming assignment. It includes all the necessary information about the assignment, the algorithms implemented, the parameters used, and the results obtained. Furthermore, it outlines the challenges that had to be overcome during the development process.

2 Overview

In this programming exercise, we are tasked with developing construction and search heuristics for the *Minimum Weighted Crossing with Constraints Problem (MWCCP)*. In brief, this involves working with a bipartite graph $G = (U \cup V, E)$, where the goal is to minimize the sum of weights of the crossed edges while maintaining constraints such that that vertex v must appear before vertex v'. To achieve this, we aim to find a permutation π of the vertices V such that the objective function is minimized:

$$f(\pi) = \sum_{(u,v)\in E} \sum_{(u',v')\in E \; u<u'} (w_{u,v} + w_{u',v'}) \cdot \delta_\pi((u,v),(u',v'))$$

Here, $\delta_\pi(\cdot,\cdot)$ is an indicator function that returns 1 if the two edges cross in the solution defined by the permutation π, and 0 otherwise. Formally, it is defined as:

$$\delta_\pi((u,v),(u',v')) = \begin{cases} 1, & \text{if } \mathrm{pos}_\pi(v) > \mathrm{pos}_\pi(v'), \\ 0, & \text{otherwise.} \end{cases}$$

To achieve this, we implemented the following algorithms:

- **Construction Heuristics**: We developed a greedy algorithm that can operate either deterministically or randomly, depending on the parameter k, which controls the size of the Restricted Candidate List (RCL). Additionally, we combined the greedy algorithm with a local search to create a GRASP (Greedy Randomized Adaptive Search Procedure) algorithm.
- **Search Heuristics**: We implemented a local search (LS) approach, which includes three neighborhood structures. Moreover, we utilized these structures to implement a Variable Neighborhood Descent (VND) algorithm.
- **Metaheuristics**: We implemented Simulated Annealing (SA) as our metaheuristic approach.

These algorithms will be described in greater detail in the following sections.

2.1 Applications

While problems like MWCCP are interesting in theory, it is equally important to understand their practical applications. For this reason, we explore real-world scenarios where solving MWCCP can provide real benefits.

One such example would be **Circuit Board Design**: As discussed in the lecture, this problem can be applied to circuit board design, where there is a mapping between two types of components. Crossings between wires or connectors can lead to electromagnetic interference, which degrades signal quality. To avoid costly production processes, minimizing these crossings, weighted by the importance of the connections between components, can significantly improve design efficiency and functionality.

3 Preprocessing

The goal of our preprocessing was to lay a foundation that can later be built upon with the greedy Construction Heuristic and the Delta Evaluation.

Before delving into the specific values we chose to precompute, we first want to illustrate the structure of a solution. A solution can be understood as a permutation of the nodes in the set V, and the objective value is influenced by this permutation. Therefore, rather than considering edges between the nodes in sets U and V, we can instead focus on the impact of the ordering of nodes in V and their resulting weights. This assumption is valid because the decision of whether two outgoing edges from two distinct nodes, $v_1, v_2 \in V$, will cross depends solely on whether v_1 precedes v_2 in the ordering, not on their exact positions in the solution.

Based on this observation, we decided to precompute the partial objective values resulting from placing node v_1 before node v_2 in the solution, as well as the partial objective values when the order is reversed. This approach reduces the complexity from depending on the number of edges $|E| = m$ to depending on the number of nodes $|V| = n$.

$$w'_{v_1 v_2} = \sum_{(v_1,u_1)\in E} \sum_{(v_2,u_2)\in E} (w_{u_1,v_1} + w_{u_2,v_2}) \cdot \gamma(u_1, u_2)$$

$$\gamma(u_1, u_2) = \begin{cases} 0, & \text{if } u_1 \leq u_2, \\ 1, & \text{if } u_1 > u_2 \end{cases}$$

4 Construction Heuristics

Based on the aforementioned precomputed partial objective values we opted to rank the nodes in V by the following scores. The greedy constuction algorithm then simply picks the node with the lowest score that is currently available, also taking the constraints into account. The construction can later be randomized by simply picking a node randomly out of the best performing nodes.
The scores we used:

$$score(v) = \frac{1}{deg(v)} \sum_{v_2 \in V, v_2 \neq v} w'_{v v_2}$$

5 Search Heuristics

When implementing the search heuristics, we followed the algorithms presented in the lecture without any modifications. Notably, we did not use any external frameworks. Instead, we developed a lightweight framework tailored to meet the requirements specified in the exercise description.

In summary, our algorithms are implemented in a concise manner. Each algorithm is provided with the corresponding neighborhood function and step function. For the stopping criteria, we opted to use a maximum runtime for LS. This makes runtimes feasible and comparable across different instances, as opposed to an iteration-based stopping criterion. The only exception is when no improvement can be found, in which case the search terminates even before the maximum runtime is reached.

5.1 Neighborhood Structures

Neighborhood structures are represented by an iterator interface that generates the next move along with the respective cost of the move (using delta-evaluation). Additionally, it includes a method to apply the move to the solution. We decided to implement the following three neighborhood structures:

- **Swap-Neighborhood**: In this neighborhood, we swap two vertices $\pi = \{..., v_i, v_{i+1}, ...\}$ with each other, resulting in a new permutation $\pi' = \{..., v_{i+1}, v_i, ...\}$. The runtime of generating this solution space is $O(|V|)$.
- **Insert-Neighborhood**: In this neighborhood, we take a vertex v_i within $\pi = \{..., v_{i-1}, v_i, v_{i+1}, ...\}$ and insert it into another position j $(i \neq j)$ within the boundaries of the constraints in the permutation, to result in $\pi = \{..., v_{i-1}, v_{i+1}, ..., v_j, v_i, ...\}$. The runtime of generating this solution space is $O(|V| \times |V|)$.
- **Exchange-Neighborhood**: In this neighborhood, we swap two vertices v_i and v_j $(i \neq j)$ within the constraint boundaries of the permutation, such that: $\pi = \{..., v_{i-1}, v_i, v_{i+1}, ..., v_{j-1}, v_j, v_{j+1}, ...\}$ becomes $\pi' = \{..., v_{i-1}, v_j, v_{i+1}, ..., v_{j-1}, v_i, v_{j+1}, ...\}$. The runtime of generating this solution space is $O(|V| \times |V|)$.

5.2 Step Functions

We implemented first, best, and random improvement. Each is implemented as a loop that iterates through the neighborhood iterator until the conditions for first, best, or random improvement are met. It then returns the next move and respective costs. This approach ensures a very fast evaluation for first or random improvement, as costs are calculated on demand rather than for all moves simultaneously. Additionally, we ensured that the entire search space was not precomputed at each step. Instead, we generated only the necessary portions and expanded the search space incrementally as needed.

5.3 Delta evaluation

One key observation regarding any move performed within the permutation is as follows: When swapping two vertices, all edges that were previously crossed will no longer be crossed, and all edges that were not crossed will now become crossed. This observation extends to the neighborhood structures mentioned earlier, where we must account for all vertices affected by the respective swap, insertion, or exchange move.

Using this observation in combination with our pre-processing approach, we can efficiently calculate the cost of a move. Instead of recomputing the objective function from scratch, we remove the costs of all combinations of vertex pair orderings affected by the move and add the costs of the new combinations. This enables a delta evaluation in $O(|V|)$, rather than recalculating the objective value for each move, which would take $O(|E| \times |E|)$. This drastically improves computational performance.

5.4 VND & SA

The implementations of **VND** and **SA** follow the same structure as the algorithms discussed above. They utilize the same techniques developed during the implementation of the LS.

The only differences are as follows:

- **VND**: This algorithm operates on a list of neighborhoods, systematically exploring each one in a predefined sequence to find an improved solution.
- **SA**: This algorithm requires additional parameters, including the initial temperature (T_{init}), a cooling function, and an equilibrium condition. These parameters are essential to ensure the proper functioning of the algorithm.

6 Experimental Setup

We conducted the following experiments on the *Conan* server of the AC group's computing cluster. While some experiments for different instances where of course conducted on different cores, each run on its own was of course not multithreaded to enable a fair comparison.

Specification	Value
Model Name	AMD Ryzen 9 5900X 12-Core Processor
CPU Architecture	x86_64
Cores	12
Base/Max Speed	2.2 GHz / 3.7 GHz

Table 1: *Conan* CPU Specifications

Hyperparameter tunig was done by hand on the smaller instances while developing the algorithms and was therefore omited from the report for simplicity. We finally settled for the following hyperparamter for the different algorithms.

Random Seed: Each run was randomized by the its individual run number. Additionally to ensure controllable randomization we first opted to pass this random seed further down to initialize the individual parts of the algorithms. However this resulted in some problems, resulting in the same randomized searches between different runs, which is why we replaced all seeds with a random seed generated at runtime and seeded by the initial seed of the run.

Number of Runs: To ensure some statistical significance of the results we opted to rerun each of the experiments at least 30 times, as this was a number recomended by Prof. Günther Raidl in one of our previous project. However due to the very large size of some of the instances, we had to reduce this number to 10 for all instances with 500 or more nodes.

Runtime: Due to the vast differences in size between the instances we decided to determine the maximum runtime based on the number of nodes contained in an instance. We employed the formula $\frac{|V|}{50}$ to determine the number of seconds an algorithm got to compute a solution. This was not ideal for each algorithm, since some algorithms like Simulated Annealing and GRASP had difficulties to get compute enough iterations on the bigger graphs, however the algorithms that struggled the most where the some local searches using the Best Improvement Step Function, as discussed later in detail.

k - Size of Restricted Candidate List: The Restricted Candidate List (RCL) employed in the randomized Greedy and the GRASP algorithm was set to 10 as this proofed to be a well balanced middleground between having an completely randomized algorithm and a to narrow algorithm, that would hinder especially GRASP to explore as much of the searchspace as possible in the given time.

Simulated Annealing Parameter For SA, we used the following parameters to run the experiments:

- **T_INIT=**$10000 \cdot \left(\frac{|V|}{50}\right)^2$: The initial temperature for each SA run. We derived this formula, $T_{\text{init}} = f_{\max} - f_{\min}$, based on observations from experimental runs. The formula adjusts depending on the problem size to ensure a reasonable starting temperature.

- **NUM_ITER_T_NOT_IMPROVED=10**: The number of levels at which different temperature values (T) did not result in any improvement. This acts as a stopping criterion for the annealing process.

- **COOLING_SCHEDULE="geometric"**: We used a geometric cooling strategy to calculate the temperature at each step.

- **COOLING_SCHEDULE_ALPHA=0.95**: The cooling rate α for the geometric cooling strategy. After conducting several tests, we found that $\alpha = 0.95$ provided good results in balancing exploration and convergence.

- **ITER_EQ_COND="gen_based_nodes"**: We defined the number of iterations to satisfy the equilibrium condition as $|V| \times (|V| - 1)$, similar to the approach discussed in the lecture. This allows us to cover most of the search space of the Insert-Neighborhood.

7 Results

Next, we continue by presenting the results of the experiments. First, all results concerning runtimes, average objective values, and standard deviations can be found in the tables in Section 8.

Let us start with the evaluation of the construction heuristics and GRASP. As shown in Table 2, the deterministic construction heuristic performs quite well. In fact, due to its low running times, LS only improves the results marginally for some instances. Furthermore, even for the largest instances, the runtime is only 0.21 seconds, which is very efficient. The randomized construction heuristic, on the other hand, performs slightly worse, with a significantly higher standard deviation for most instances. However, its performance is not significantly worse compared to the deterministic heuristic. As expected, the runtime of the randomized heuristic is as fast as the deterministic version. GRASP performs very well, consistently outperforming the deterministic heuristic for all instances, with moderate running times. The impact of the local search becomes more evident with increasing instance size, as the number of GRASP iterations decreases from over 100 for smaller instances to around 2–3 for the largest ones. However, increasing the maximum runtime could potentially improve GRASP's performance for larger instances.

Continuing with the performance of LS, Table 3 shows the results for the Swap neighborhood. First, we observe that while we do not achieve perfect results—for example, in the instance `comp_50_4_00001`—the low running times make this neighborhood attractive. Even for large instances, we manage to perform approximately 5000–7000 iterations in just 1–5 seconds using either best or first improvement. The low complexity of $O(|V|)$ proves highly advantageous in this context, making Swap an optimal choice for use in the GRASP algorithm.

In contrast, the performance of the Insert neighborhood, while producing optimal solutions (according to the benchmark competition), is hindered by its larger neighborhood size of $O(|V| \times |V|)$. Table 4 shows that, in combination with the best improvement step function, this neighborhood requires significant runtime, particularly for larger instances. For instance, with instances containing 1000 nodes, we observe that only one iteration is completed on average, while the runtime exceeds the time limit by 20 seconds. In comparison, the first improvement step function manages to achieve an average of 13,500 iterations across instances with 1000 nodes.

Interestingly, the performance of the Exchange neighborhood, shown in Table 5, demonstrates a more balanced runtime despite the equally large search space. Here, the runtime remains within acceptable bounds, allowing 20 times as many iterations for best improvement and 3 times as many for first improvement compared to Insert. While the Exchange neighborhood performs slightly worse than Insert in terms of solution quality, it compensates by exploring significantly more solutions within the same timeframe. As a result, even though it sometimes yields suboptimal solutions, its ability to evaluate a larger number of solutions makes it a viable alternative.

Considering runtimes in Figure 1, we observe that for most instances and all combinations of step functions and neighborhood functions, the algorithms remain within the runtime boundaries. In most cases, local optima are found before reaching the runtime limit. The only exception is for the largest instances using the Insert neighborhood with the best improvement step function. As previously mentioned, these runs exceed the boundary by approximately 20 seconds. This makes the decision for the optimal neighborhood function challenging, as the Insert neighborhood produces high-quality solutions but at the cost of significantly longer runtimes for large instances.

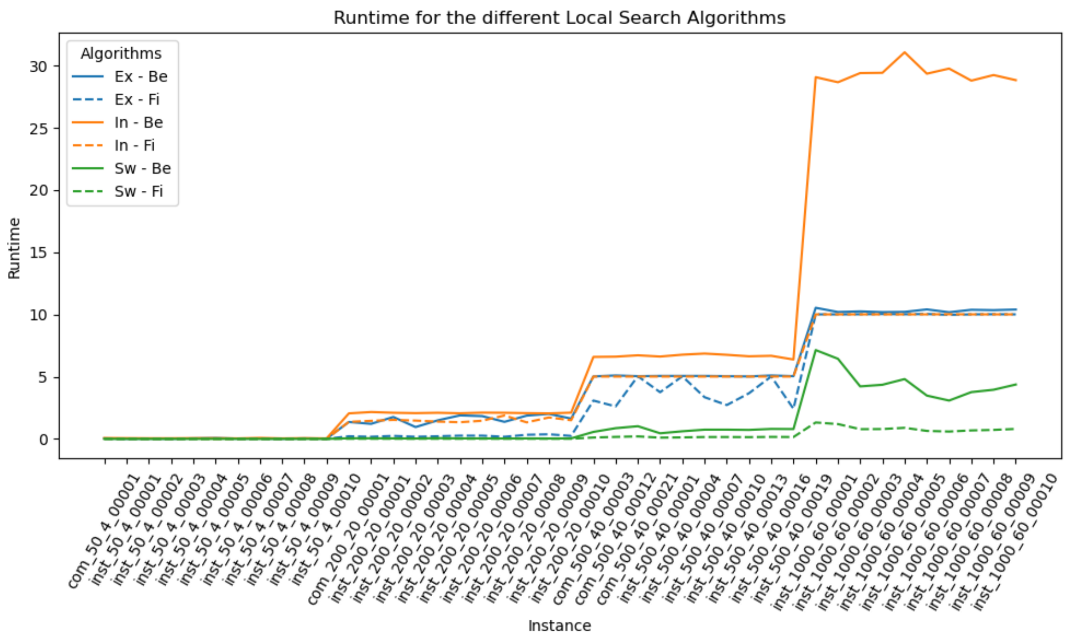

Figure 1: Comparison between the different Local Search Runtimes

Randomization within neighborhoods and algorithms is essential to generate a diverse set of solutions that can be meaningfully compared. Incorrect implementation of randomization can lead to similar or even erroneous results, as we encountered during the development of this code. Furthermore, ensuring reproducibility—a critical aspect of scientific research—adds another layer of complexity. We only noticed a problem with this procedure minutes before the final runs, highlighting the challenges of achieving perfection in this aspect.

Lastly, considering the step functions, it is evident that first improvement is the overall winner. It achieves comparable or even better results than best improvement while requiring less time and allowing more iterations. This is particularly evident in Table 4, which evaluates the Insert neighborhood. Notably, the performance is due to the fast delta-evaluation. Without it, the runtimes with each step function and neighborhood function would not be feasible and, therefore, not usable. Additionally, we omitted any evaluation of the random step function, as it was not expected to yield good results. This assumption is validated by the results in all LS tables, where the random step function consistently performs worse.

Looking at Table 6, VND performs the best among all algorithms. It achieves very good results in minimal time, requiring only a small number of iterations between neighborhoods.

For VND, we used the neighborhoods in the following order: Swap, Insert, and Exchange. This order was chosen under the assumption that it would evenly separate the solution space and consistently return improvements. However, after conducting the experiments, we concluded that the order should have been Swap, Exchange, and Insert. This adjustment would better handle larger solution spaces by prioritizing neighborhoods with smaller search spaces first and leaving the more computationally expensive neighborhoods, like Insert, for last. Such a change could potentially improve both the runtime and the overall performance of VND.

Again, looking at Table 6, we can observe that SA performs poorly compared to VND and other LS results. One key reason for this is the challenge of finding optimal hyperparameters for different instances and sizes, which is crucial for benchmarking and comparison. Despite our efforts, there is still room for improvement in this area. Additionally, time plays a critical role in the performance of SA. As shown in Figure 2, the solution steadily improves over time. However, SA could benefit from more iterations and, consequently, more runtime to converge to a truly optimal solution. This limitation highlights the trade-off between runtime and solution quality when using SA.

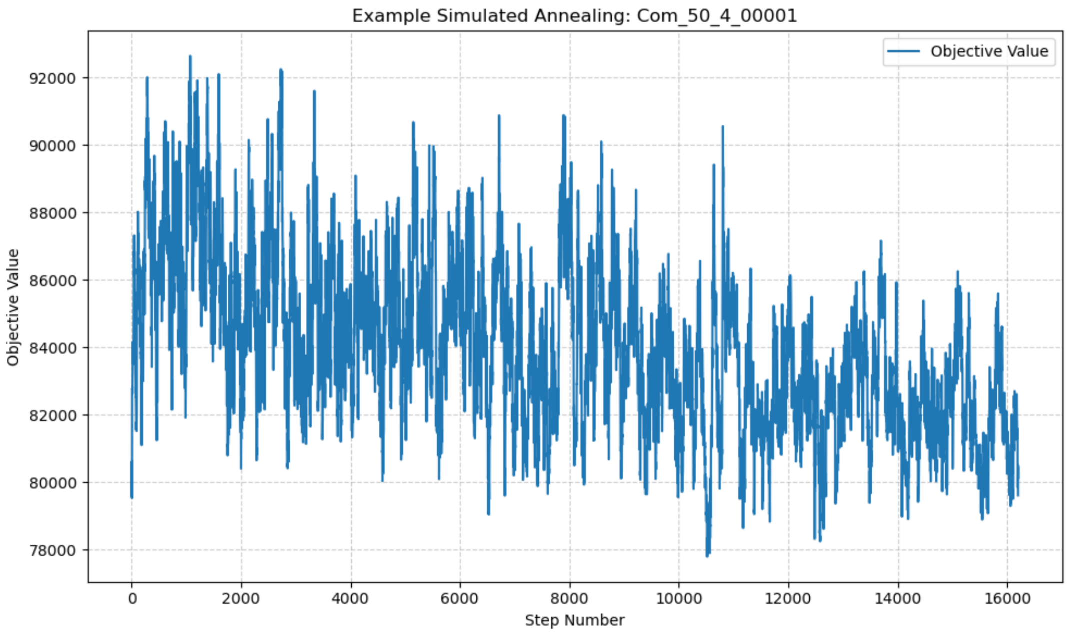

Figure 2: Example Run for Simulated Annealing

Also considering the results in Table 6, for the largest and second-largest instances, we observe that all runs have the same number of iterations. This is due to the equilibrium condition, which allows covering only one temperature level because of the limited runtime. With more time, SA would likely converge and produce better results. However, due to the computational and time constraints of this exercise, this was not feasible. This observation aligns with the pattern discussed during the lecture: LS can outperform SA for certain problems, particularly when runs are time-bound.

To compare the best solution found for each algorithm configuration on each instance, you can take a look at the tables: Table 7, Table 8 and Table 9.

Lastly, considering the benchmark results from the TUWEL course at 21:00 on 24.11.2024, we observe that our approach generates the best or near-best results within a runtime limit of only 10 seconds per instance. This demonstrates the effectiveness of our implementation. It would be interesting, however, to see whether longer runtimes—such as several minutes or more—would yield the absolute best solutions for each instance.

8 Conclusion

In conclusion, we find that Greedy and GRASP work well for this problem, even when using randomized approaches. Pre-processing and efficient delta evaluation are key to success in this context. Performing delta evaluations with a complexity of $O(|E| \times |E|)$ is not feasible, especially for large instances. Therefore, drafting an efficient delta evaluation strategy before implementing the algorithms was the most critical step.

We also observed that different neighborhoods converge at varying speeds toward optimal solutions. However, a larger neighborhood space does not automatically guarantee better solutions. For this reason, selecting the right step function is crucial. In our case, considering exploration rate, runtime, and performance, the first improvement step function is the clear winner.

Lastly, combining neighborhood structures in VND performs the best, as expected, while SA struggles due to its dependency on longer runtimes, especially for large instances. Within the time constraints of this exercise, SA could not fully demonstrate its potential. Overall, while finding significant improvements and optimal solutions is challenging, techniques like those we used in this exercise make this task more manageable.

9 Tables

This section contains all the tables generated after all runs.

	Greedy Deterministic		Greedy Random k=10			GRASP k=10				
	μ_{obj}	t	μ_{obj}	σ_{obj}	t	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}
com_50_4_00001	79610.00	0.00	84638.03	1919.90	0.00	76287.60	15.21	0.50	125.50	2.08
inst_50_4_00001	75307.00	0.00	81190.80	1736.79	0.00	74310.37	24.28	0.50	174.67	4.33
inst_50_4_00002	24569.00	0.00	27391.87	585.51	0.00	23982.20	0.81	0.50	143.10	3.20
inst_50_4_00003	12590.00	0.00	14156.60	429.17	0.00	12298.90	9.96	0.50	165.53	4.48
inst_50_4_00004	7284.00	0.00	8623.80	449.54	0.00	6667.27	36.06	0.50	154.80	4.02
inst_50_4_00005	3923.00	0.00	4766.60	226.94	0.00	3582.43	1.89	0.50	147.87	4.34
inst_50_4_00006	3131.00	0.00	3935.10	233.05	0.00	2963.43	2.73	0.50	158.67	3.06
inst_50_4_00007	2588.00	0.00	3295.67	212.24	0.00	2281.23	40.63	0.50	315.77	8.00
inst_50_4_00008	1513.00	0.00	2234.17	192.31	0.00	1465.50	15.45	0.50	283.30	6.79
inst_50_4_00009	1489.00	0.00	1995.33	152.09	0.00	1381.27	21.01	0.50	313.43	8.08
inst_50_4_00010	1227.00	0.00	1492.70	168.61	0.00	894.57	14.76	0.50	280.30	6.01
com_200_20_00001	21775940.00	0.01	21905954.07	45431.17	0.01	21543087.87	7919.75	2.00	25.60	0.62
inst_200_20_00001	22060987.00	0.01	22182549.97	39751.10	0.01	21829114.70	6565.48	2.00	24.97	0.49
inst_200_20_00002	7738678.00	0.01	7797458.70	21890.54	0.01	7646438.60	3110.36	2.00	28.80	0.41
inst_200_20_00003	3889565.00	0.01	3930307.20	13133.32	0.01	3854399.53	3069.47	2.00	32.47	0.73
inst_200_20_00004	2258014.00	0.01	2284812.60	8559.11	0.01	2211795.13	1280.17	2.00	24.93	0.52
inst_200_20_00005	1394712.00	0.01	1412550.13	8022.29	0.01	1370449.43	765.90	2.00	27.53	0.63
inst_200_20_00006	1055928.00	0.01	1076738.97	7983.55	0.01	1028770.90	1857.65	2.00	29.77	0.63
inst_200_20_00007	766948.00	0.01	781198.40	5422.53	0.01	751139.73	596.81	2.00	33.17	0.65
inst_200_20_00008	577661.00	0.01	588514.13	3763.45	0.01	564552.70	453.38	2.00	32.30	0.60
inst_200_20_00009	453058.00	0.01	461060.17	2629.83	0.01	443743.33	376.89	2.00	37.50	0.57
inst_200_20_00010	371300.00	0.01	377726.53	2878.69	0.01	358847.90	451.61	2.00	31.57	0.73
com_500_40_00003	68443048.00	0.05	68557105.63	40087.19	0.05	67911928.35	17173.22	5.00	7.48	0.50
com_500_40_00012	288783726.00	0.05	289105469.70	103192.37	0.05	286456725.17	40920.26	5.00	5.22	0.42
com_500_40_00021	587513266.00	0.05	588115239.00	291971.23	0.05	583376836.80	307035.78	5.00	5.00	0.00
inst_500_40_00001	37374937.00	0.05	37473693.70	29880.45	0.05	37067487.07	16170.04	5.01	8.40	0.50
inst_500_40_00004	86510996.00	0.05	86676654.73	56571.53	0.05	85731517.17	29161.74	5.00	7.00	0.00
inst_500_40_00007	155924541.00	0.05	156179086.93	97289.71	0.05	154545776.37	33010.64	5.00	6.00	0.00
inst_500_40_00010	233475415.00	0.05	233814791.90	98026.57	0.05	231701775.57	51699.66	5.00	6.00	0.00
inst_500_40_00013	317406799.00	0.05	317808707.93	143860.92	0.05	315445147.70	54006.18	5.00	6.03	0.18
inst_500_40_00016	417182294.00	0.05	417614752.77	176140.73	0.05	414483360.10	96664.61	5.00	5.67	0.48
inst_500_40_00019	512123351.00	0.05	512590112.53	181114.95	0.05	508519577.90	82996.44	5.00	5.00	0.00
inst_1000_60_00001	14823503220.00	0.21	14826410249.10	684521.05	0.21	14769576395.90	1687105.61	10.00	2.00	0.00
inst_1000_60_00002	5171232271.00	0.20	5172776607.30	458563.93	0.21	5147645173.00	712607.93	10.00	2.00	0.00
inst_1000_60_00003	2579838354.00	0.21	2581201397.20	348031.28	0.21	2569766890.00	537075.63	10.02	2.50	0.53
inst_1000_60_00004	1529974118.00	0.21	1530676725.80	297156.12	0.21	1522966136.90	199507.50	10.00	2.80	0.42
inst_1000_60_00005	1007405055.00	0.22	1007914471.90	133458.30	0.22	1001739136.00	243443.29	10.00	2.00	0.00
inst_1000_60_00006	719034385.00	0.21	719492094.20	80864.37	0.21	716087557.60	94652.62	10.00	3.00	0.00
inst_1000_60_00007	526150590.00	0.21	526475258.30	94455.91	0.21	523598789.20	90508.62	10.00	3.00	0.00
inst_1000_60_00008	412916980.00	0.21	413230984.30	81366.63	0.21	410228957.70	86981.07	10.00	3.00	0.00
inst_1000_60_00009	327382939.00	0.21	327590822.20	26434.55	0.21	325364846.20	30071.87	10.00	3.00	0.00
inst_1000_60_00010	268843223.00	0.21	269064805.60	54753.86	0.21	266802500.60	56116.28	10.06	2.70	0.48

Table 2: Results of the Greedy Construction and GRASP

	Best Improvement					First Improvement					Random Improvement				
	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}
com_50_4_00001	76653.00	0.00	0.00	53.00	0.00	76653.00	0.00	0.00	53.00	0.00	88168.43	1676.47	0.50	48479.83	166.02
inst_50_4_00001	74906.00	0.00	0.00	12.00	0.00	74919.33	6.06	0.00	12.83	0.38	84759.33	2344.52	0.50	48292.53	160.91
inst_50_4_00002	24205.00	0.00	0.00	18.00	0.00	24263.13	29.12	0.00	16.40	0.81	29723.40	993.14	0.50	45766.67	512.13
inst_50_4_00003	12315.00	0.00	0.00	23.00	0.00	12325.77	8.33	0.00	21.10	1.47	15670.50	695.54	0.50	46722.47	448.00
inst_50_4_00004	6856.00	0.00	0.00	30.00	0.00	6857.87	3.15	0.00	30.00	0.00	9123.83	528.03	0.50	47338.73	152.70
inst_50_4_00005	3599.00	15.26	0.00	34.50	2.54	3607.90	10.96	0.00	33.57	1.81	5371.57	438.51	0.50	45605.83	153.45
inst_50_4_00006	3049.00	0.00	0.00	9.00	0.00	3046.80	1.35	0.00	9.73	0.45	4548.47	217.47	0.50	46396.67	119.66
inst_50_4_00007	2530.00	0.00	0.00	15.00	0.00	2530.00	3.94	0.00	15.00	0.79	3724.30	353.35	0.50	46787.93	318.89
inst_50_4_00008	1456.00	0.00	0.00	11.00	0.00	1453.87	2.03	0.00	12.07	1.01	2754.13	286.29	0.50	46248.30	314.49
inst_50_4_00009	1433.00	0.00	0.00	11.00	0.00	1433.00	0.00	0.00	11.00	0.00	2445.60	229.23	0.50	46949.17	199.85
inst_50_4_00010	1170.00	0.00	0.00	9.00	0.00	1170.00	0.00	0.00	9.00	0.00	1636.43	141.90	0.50	46863.07	372.48
com_200_20_00001	21594503.00	0.00	0.05	453.00	0.00	21596592.00	2132.46	0.01	448.00	5.09	22632966.97	146136.76	2.00	83680.80	982.86
inst_200_20_00001	21864987.20	85.70	0.05	432.60	0.50	21864962.63	82.46	0.01	431.90	0.96	22831197.23	110977.00	2.00	84503.73	185.28
inst_200_20_00002	7658818.00	0.00	0.04	386.00	0.00	7660193.50	1110.97	0.01	375.63	6.67	8134080.63	69732.32	2.00	82525.73	258.07
inst_200_20_00003	3861909.80	36.37	0.03	274.20	1.49	3861853.33	48.61	0.01	276.23	1.48	4155942.77	37007.06	2.00	81941.33	261.45
inst_200_20_00004	2215649.93	44.65	0.06	481.00	0.00	2216177.23	621.95	0.01	468.53	10.89	2422253.67	27837.37	2.00	82833.13	272.54
inst_200_20_00005	1376115.33	6.91	0.04	353.07	0.69	1376864.43	390.12	0.01	341.10	5.68	1521845.13	14984.79	2.00	81299.33	389.57
inst_200_20_00006	1033153.40	305.95	0.04	371.03	3.65	1033413.83	522.98	0.01	369.40	9.87	1147684.93	15536.80	2.00	81963.87	225.75
inst_200_20_00007	753009.00	0.00	0.03	294.00	0.00	752917.03	86.99	0.01	302.07	3.23	850535.93	16326.54	2.00	82528.73	236.20
inst_200_20_00008	565453.00	0.00	0.04	348.00	0.00	564937.50	324.56	0.01	354.87	5.53	664595.80	15288.56	2.00	82600.33	263.71
inst_200_20_00009	445665.90	1.40	0.03	244.00	0.00	445629.47	157.40	0.01	245.40	4.81	514091.57	10248.83	2.00	82151.53	193.46
inst_200_20_00010	360979.83	2.52	0.04	347.87	1.01	361388.37	403.97	0.01	336.63	13.89	420480.70	7230.83	2.00	81295.77	364.82
com_500_40_00003	67926405.00	0.00	0.56	1950.00	0.00	67925336.23	3028.89	0.11	1946.73	12.77	69012073.87	103736.19	5.00	99279.77	608.28
com_500_40_00012	286597763.00	0.00	0.86	2868.00	0.00	286613032.20	10625.38	0.16	2824.73	21.37	290131777.30	276288.63	5.00	98447.27	438.95
com_500_40_00021	583214450.00	0.00	1.02	3477.00	0.00	583215667.40	1146.66	0.20	3471.77	5.93	589319880.60	501797.33	5.00	97146.53	357.83
inst_500_40_00001	37126642.00	0.00	0.45	1547.00	0.00	37126937.13	2115.41	0.10	1528.50	17.09	37734838.63	85939.39	5.00	96771.80	2421.30
inst_500_40_00004	85813232.20	6.05	0.61	2061.57	0.50	85810078.40	4789.12	0.12	2068.90	17.93	87249784.43	123354.82	5.00	99619.47	383.22
inst_500_40_00007	154583375.00	0.00	0.74	2499.00	0.00	154584944.97	1687.77	0.15	2495.50	7.15	156896818.07	230125.56	5.00	96832.03	358.71
inst_500_40_00010	231829044.00	0.00	0.74	2485.00	0.00	231828253.00	6672.68	0.15	2490.80	13.78	234844120.60	285322.25	5.00	99522.77	281.86
inst_500_40_00013	315589596.00	0.00	0.72	2420.00	0.00	315596456.87	6004.82	0.14	2401.43	14.51	318877760.70	307393.55	5.00	98771.10	465.08
inst_500_40_00016	414681252.00	0.00	0.80	2715.00	0.00	414674350.10	5837.67	0.16	2737.10	14.32	418796847.70	387015.49	5.00	97635.23	262.06
inst_500_40_00019	509147033.77	130.54	0.79	2772.87	1.01	509195777.93	28981.93	0.15	2725.77	24.00	513772015.00	365946.05	5.00	101295.47	480.20
inst_1000_60_00001	14771561793.00	0.00	7.14	11372.00	0.00	14771569816.20	46581.29	1.32	11362.10	10.79	14828292331.20	1278431.50	10.00	94397.30	313.89
inst_1000_60_00002	5148864915.00	0.00	6.44	10395.00	0.00	5148942417.20	85618.40	1.19	10356.10	43.28	5174288944.10	529009.48	10.00	98334.50	349.78
inst_1000_60_00003	2570087708.00	0.00	4.22	6738.00	0.00	2570181527.30	36070.56	0.78	6649.20	28.25	2582219770.60	350938.98	10.00	92522.00	475.46
inst_1000_60_00004	1523316432.00	0.00	4.35	6814.00	0.00	1523323160.00	8432.67	0.79	6784.30	13.47	1531216471.50	331746.36	10.00	93644.10	220.96
inst_1000_60_00005	1001903829.00	0.00	4.81	7451.00	0.00	1001943330.70	65512.14	0.89	7408.10	84.56	1008525314.30	256801.61	10.00	93898.00	260.41
inst_1000_60_00006	716228682.00	0.00	3.48	5493.00	0.00	716254158.90	15643.79	0.64	5416.30	35.17	719973362.70	163256.92	10.00	92331.40	236.22
inst_1000_60_00007	523843481.00	0.00	3.08	4915.00	0.00	523684565.00	36899.26	0.59	5175.70	71.01	526810604.70	155638.56	10.00	93438.90	445.42
inst_1000_60_00008	410410307.00	0.00	3.76	6147.00	0.00	410505922.30	26937.37	0.68	5930.90	78.00	413470947.10	132120.60	10.00	93346.80	208.79
inst_1000_60_00009	325328061.60	68.01	3.95	6190.20	1.75	325369077.40	15235.81	0.72	5988.90	70.15	327890243.70	120399.59	10.00	93653.70	262.05
inst_1000_60_00010	266861119.30	53.62	4.37	6852.10	1.45	266881337.20	11641.29	0.80	6722.90	56.32	269234319.20	59791.15	10.00	92883.10	401.48

Table 3: Results of Swap Neighborhood

	Best Improvement					First Improvement					Random Improvement				
	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}
com_50_4_00001	76269.00	0.00	0.07	17.00	0.00	76269.00	0.00	0.02	44.67	5.60	86576.23	1760.12	0.50	16852.40	83.08
inst_50_4_00001	74247.00	0.00	0.06	16.00	0.00	74247.00	0.00	0.02	30.47	4.49	85994.03	1478.38	0.50	17199.87	93.88
inst_50_4_00002	23982.00	0.00	0.05	11.00	0.00	23982.00	0.00	0.01	25.07	4.71	30202.70	1354.74	0.50	16197.03	99.16
inst_50_4_00003	12278.00	0.00	0.05	12.00	0.00	12283.00	10.17	0.01	24.67	3.35	15841.70	609.28	0.50	16708.03	107.23
inst_50_4_00004	6534.00	0.00	0.06	14.53	0.51	6534.00	0.00	0.02	37.93	4.79	9183.50	571.54	0.50	17010.70	78.57
inst_50_4_00005	3580.00	0.00	0.06	12.63	0.49	3580.00	0.00	0.01	27.13	3.88	5326.40	339.69	0.50	16498.40	120.86
inst_50_4_00006	2959.00	0.00	0.04	10.00	0.00	2959.00	0.00	0.01	22.30	3.43	4488.67	288.78	0.50	16444.47	124.62
inst_50_4_00007	2042.00	0.00	0.08	20.77	0.68	2043.97	4.04	0.02	45.30	5.05	3601.57	335.63	0.50	16658.00	182.63
inst_50_4_00008	1407.00	0.00	0.04	10.00	0.00	1405.67	1.84	0.01	19.63	2.41	2669.10	281.39	0.50	16479.77	83.60
inst_50_4_00009	1295.00	0.00	0.06	14.50	0.73	1305.80	8.97	0.02	29.27	3.82	2439.07	206.56	0.50	16694.03	94.42
inst_50_4_00010	852.00	0.00	0.04	10.00	0.00	853.03	1.96	0.01	24.83	3.95	1637.60	141.30	0.50	16580.33	94.34
com_200_20_00001	21654832.07	1712.28	2.05	10.07	0.25	21459363.50	1761.71	1.37	378.23	35.29	23104755.03	214494.16	2.00	20716.43	225.14
inst_200_20_00001	21889705.00	0.00	2.16	10.00	0.00	21737464.13	443.55	1.45	326.93	18.62	23265773.57	138020.96	2.00	20978.67	123.60
inst_200_20_00002	7693198.00	0.00	2.10	10.00	0.00	7607266.57	56.66	1.53	348.63	18.26	8388745.80	81002.14	2.00	20749.63	89.19
inst_200_20_00003	3856826.00	0.00	2.07	10.00	0.00	3827466.27	87.03	1.47	277.90	14.90	4333431.10	48830.69	2.00	20716.43	129.51
inst_200_20_00004	2232247.00	0.00	2.10	10.00	0.00	2195893.43	454.46	1.39	319.30	16.09	2524932.53	35920.04	2.00	20557.20	125.68
inst_200_20_00005	1383700.00	0.00	2.06	10.00	0.00	1364834.77	36.79	1.34	282.67	15.07	1627513.30	27918.86	2.00	20476.27	114.85
inst_200_20_00006	1032317.00	0.00	2.11	10.00	0.00	998986.93	123.29	1.46	308.77	20.37	1195717.77	18325.77	2.00	20314.63	110.48
inst_200_20_00007	754404.00	0.00	2.10	10.00	0.00	736968.53	23.98	1.88	434.00	31.03	902141.27	17539.51	2.00	20601.77	113.49
inst_200_20_00008	569067.00	0.00	2.07	10.00	0.00	559806.23	16.70	1.33	274.57	15.90	722920.37	15362.23	2.00	20877.83	91.14
inst_200_20_00009	446388.40	111.89	2.05	10.10	0.31	436322.83	80.63	1.72	355.10	19.34	555414.57	13140.88	2.00	20660.77	111.97
inst_200_20_00010	363519.00	0.00	2.11	10.00	0.00	352011.13	20.37	1.50	339.23	26.03	450228.37	11760.80	2.00	20332.63	85.64
com_500_40_00003	68383595.00	0.00	6.59	2.00	0.00	67521797.67	1784.76	5.01	919.40	31.53	74695714.67	384826.49	5.00	21284.43	80.62
com_500_40_00012	288530262.00	0.00	6.61	2.00	0.00	285358554.13	32569.81	5.01	885.53	32.17	304743217.47	1054503.23	5.00	20963.40	239.73
com_500_40_00021	587102726.00	0.00	6.72	2.00	0.00	580077543.93	40996.56	5.01	1208.23	31.82	608112045.77	2133691.83	5.00	20895.07	81.97
inst_500_40_00001	37342570.00	0.00	6.62	2.00	0.00	36736354.07	6757.66	5.01	961.47	29.69	41513060.47	306966.06	5.00	21110.27	76.85
inst_500_40_00004	86429110.00	0.00	6.77	2.00	0.00	85252370.77	9054.92	5.01	908.13	31.47	94626730.70	567553.98	5.00	21087.37	50.83
inst_500_40_00007	155745644.00	0.00	6.86	2.00	0.00	153825635.47	9717.83	5.01	923.43	29.17	166204162.93	741452.14	5.00	21005.27	71.27
inst_500_40_00010	233292869.00	0.00	6.76	2.00	0.00	230788543.90	29050.42	5.01	950.93	32.48	248073474.20	1022880.10	5.00	20837.83	101.84
inst_500_40_00013	317192068.00	0.00	6.64	2.00	0.00	314307254.27	11113.94	5.01	957.90	26.69	335202030.63	1214734.24	5.00	20936.03	82.24
inst_500_40_00016	416873152.00	0.00	6.68	2.00	0.00	413018741.67	28406.51	5.01	1045.93	30.01	437018492.43	1478984.66	5.00	20951.30	74.13
inst_500_40_00019	511761046.00	0.00	6.38	2.00	0.00	507276756.70	11070.38	5.01	1032.83	31.81	533451103.30	1473931.63	5.00	21417.23	106.14
inst_1000_60_00001	14821539153.00	0.00	29.08	1.00	0.00	14732176942.50	473642.86	10.01	1551.60	28.65	15138022798.70	11685288.44	10.00	19178.40	38.62
inst_1000_60_00002	5170513120.00	0.00	28.67	1.00	0.00	5134139201.60	334709.63	10.01	1538.10	31.19	5342055792.30	10632198.89	10.00	20265.40	43.03
inst_1000_60_00003	2579273168.00	0.00	29.41	1.00	0.00	2557415141.60	156949.73	10.01	1431.90	14.78	2710153284.70	7564600.86	10.00	19098.80	46.48
inst_1000_60_00004	1529687928.00	0.00	29.43	1.00	0.00	1516068139.90	60816.13	10.01	1372.70	34.75	1619742201.50	6856118.62	10.00	19188.50	188.76
inst_1000_60_00005	1006952164.00	0.00	31.08	1.00	0.00	995737243.10	68885.95	10.01	1364.60	31.32	1069705286.90	3270788.75	10.00	18890.40	64.05
inst_1000_60_00006	718879061.00	0.00	29.36	1.00	0.00	712360020.20	47541.07	10.01	1257.90	23.47	774640446.40	1931943.56	10.00	19061.60	60.35
inst_1000_60_00007	526028126.00	0.00	29.77	1.00	0.00	520173648.10	31294.48	10.02	1268.20	23.76	568664425.70	1505353.23	10.00	19122.90	58.87
inst_1000_60_00008	412765398.00	0.00	28.80	1.00	0.00	407248030.20	32840.09	10.01	1380.60	31.98	448961197.70	1600606.99	10.00	19364.90	112.49
inst_1000_60_00009	327269610.00	0.00	29.25	1.00	0.00	322912735.40	30374.55	10.01	1302.60	28.87	357354579.20	1275732.43	10.00	19089.60	39.38
inst_1000_60_00010	268687612.00	0.00	28.84	1.00	0.00	265017865.80	50981.22	10.01	1382.80	52.61	294678415.90	1561616.36	10.00	19212.70	69.39

Table 4: Results of Insert Neighborhood

	Best Improvement					First Improvement					Random Improvement				
	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}
com_50_4_00001	76643.00	0.00	0.04	21.00	0.00	76658.80	23.56	0.02	36.00	4.06	87405.73	1946.02	0.50	5894.93	130.58
inst_50_4_00001	74931.00	0.00	0.01	6.00	0.00	74929.60	0.93	0.01	14.70	2.95	85107.80	2906.04	0.50	7466.27	151.56
inst_50_4_00002	24013.00	0.00	0.02	14.00	0.00	24011.73	37.82	0.01	24.53	4.46	29709.80	1253.46	0.50	7265.23	110.57
inst_50_4_00003	12350.00	0.00	0.01	10.00	0.00	12331.20	28.93	0.01	19.80	3.44	15263.40	544.01	0.50	7584.37	123.42
inst_50_4_00004	6856.00	0.00	0.04	17.00	0.00	6837.33	57.27	0.01	24.37	5.31	8635.40	449.40	0.50	6024.73	152.99
inst_50_4_00005	3594.00	0.00	0.07	21.00	0.00	3592.50	4.73	0.03	30.13	4.07	5435.60	348.72	0.50	6896.60	105.31
inst_50_4_00006	2990.27	11.67	0.02	12.87	1.01	2991.83	17.49	0.01	19.87	4.14	4530.37	293.45	0.50	6870.47	93.25
inst_50_4_00007	2485.83	31.09	0.03	12.07	2.73	2373.00	105.48	0.01	25.17	6.12	3969.53	308.58	0.50	7264.60	120.81
inst_50_4_00008	1440.47	19.15	0.01	8.13	1.01	1439.37	15.71	0.01	12.80	2.82	2504.60	257.67	0.50	7023.50	106.67
inst_50_4_00009	1385.63	2.14	0.04	13.77	1.22	1384.90	11.62	0.01	17.70	1.78	2352.57	201.16	0.50	7479.67	104.24
inst_50_4_00010	888.53	4.03	0.02	16.00	0.00	907.97	33.03	0.01	30.37	7.00	1739.93	119.26	0.50	7081.73	94.98
com_200_20_00001	21530406.00	0.00	1.36	156.00	0.00	21570504.57	14132.46	0.21	318.07	20.72	23082169.50	182128.13	2.00	9267.13	441.13
inst_200_20_00001	21853052.00	0.00	1.21	150.00	0.00	21840639.50	12702.73	0.17	302.17	17.12	23219579.03	136516.26	2.00	9476.23	384.95
inst_200_20_00002	7639504.00	108.00	1.76	169.83	0.91	7648148.87	5730.55	0.24	296.03	24.26	8399801.53	89956.35	2.00	8711.97	423.75
inst_200_20_00003	3857004.00	0.00	0.95	112.00	0.00	3857323.73	1836.90	0.17	223.73	18.90	4334465.23	52453.09	2.00	9272.53	326.70
inst_200_20_00004	2208470.00	0.00	1.49	150.00	0.00	2207280.57	2204.08	0.21	357.30	25.09	2540022.57	29108.94	2.00	9162.17	428.28
inst_200_20_00005	1369666.90	8.81	1.89	157.53	1.55	1371080.03	1962.95	0.26	296.40	28.41	1631707.87	30197.82	2.00	8997.10	313.44
inst_200_20_00006	1021812.53	1.96	1.83	157.37	0.49	1021110.10	2222.33	0.26	359.50	31.47	1194093.07	17009.22	2.00	9226.33	258.72
inst_200_20_00007	750259.80	248.73	1.37	112.87	2.70	751323.77	2111.24	0.17	264.70	27.64	900449.07	16825.21	2.00	8578.47	274.58
inst_200_20_00008	561866.00	0.00	1.87	113.00	0.00	562923.73	720.29	0.33	261.23	17.75	723327.33	17575.83	2.00	9548.30	363.36
inst_200_20_00009	443448.30	99.18	2.01	96.90	1.09	442626.70	663.75	0.37	250.17	21.36	554324.33	11919.53	2.00	8637.13	486.10
inst_200_20_00010	358071.87	40.31	1.63	135.43	3.45	355899.40	1187.62	0.24	348.80	34.24	453881.87	11413.08	2.00	8278.70	330.72
com_500_40_00003	68119175.57	599.43	5.02	84.37	0.49	67734131.67	30289.10	3.08	1215.50	53.00	73713461.33	312507.23	5.00	7914.27	522.26
com_500_40_00012	288010195.80	5939.81	5.10	31.53	0.51	286265189.93	76262.22	2.61	1253.43	71.48	301775570.07	1193921.35	5.00	8630.30	641.16
com_500_40_00021	586110562.00	0.00	5.04	39.00	0.00	582338932.37	286032.50	5.04	1587.83	112.79	604163002.47	1737729.33	5.00	9205.67	628.47
inst_500_40_00001	37204977.77	529.56	5.06	44.23	0.43	36975985.37	37436.49	3.75	1094.23	102.30	40735536.33	242982.43	5.00	9164.33	625.60
inst_500_40_00004	86180854.00	0.00	5.06	32.00	0.00	85619765.53	48560.45	5.03	1097.43	77.03	93040971.33	598671.06	5.00	8516.70	551.43
inst_500_40_00007	155277151.50	1374.60	5.06	53.90	0.31	154338556.90	59815.95	3.35	1448.20	90.36	164625224.00	778388.73	5.00	8873.27	582.21
inst_500_40_00010	232565031.00	7006.40	5.04	59.33	0.48	231525602.97	56716.62	2.72	1290.80	69.75	245366178.13	1073221.10	5.00	8570.80	649.63
inst_500_40_00013	316404935.60	2718.74	5.02	52.07	0.25	315102024.67	129186.04	3.67	1300.20	100.57	332205114.93	1490412.90	5.00	8725.97	758.96
inst_500_40_00016	416482137.00	0.00	5.10	23.00	0.00	414371905.60	141087.22	5.02	1209.30	74.64	432972924.73	1381883.08	5.00	8870.07	654.53
inst_500_40_00019	510577526.67	9549.17	5.04	74.10	0.40	508201251.93	437346.14	2.42	1527.70	135.52	529037654.23	1854957.51	5.00	9023.03	696.49
inst_1000_60_00001	14816845989.00	0.00	10.54	21.00	0.00	14751824278.30	2559619.19	10.01	3709.30	254.19	14947285074.20	7132690.23	10.00	8341.10	608.10
inst_1000_60_00002	5168177323.00	0.00	10.20	23.00	0.00	5141425843.50	1103487.05	10.01	3883.60	240.02	5231266571.20	5476292.17	10.00	7590.70	563.39
inst_1000_60_00003	2578308315.00	23117.30	10.25	22.50	0.53	2566645162.20	867961.93	10.02	3174.40	195.96	2628192911.30	4485403.91	10.00	7910.20	667.16
inst_1000_60_00004	1528729192.00	0.00	10.19	40.00	0.00	1519848448.70	469693.54	10.04	3594.00	194.32	1562088908.90	2711828.12	10.00	8827.50	963.00
inst_1000_60_00005	1006307441.00	0.00	10.21	39.00	0.00	998532525.90	316365.52	10.02	3661.20	159.08	1032485346.50	2098532.93	10.00	6766.50	436.76
inst_1000_60_00006	718271571.00	0.00	10.41	26.00	0.00	715238584.40	267703.27	10.04	2544.80	245.90	741118580.70	2157862.63	10.00	7089.80	630.88
inst_1000_60_00007	525550588.00	0.00	10.18	32.00	0.00	522289482.40	185692.09	9.97	3310.10	257.81	542795712.20	1584356.62	10.00	8419.10	685.53
inst_1000_60_00008	412498066.50	8216.12	10.38	15.50	0.53	409139690.60	279689.48	10.01	3139.80	310.23	427000098.80	1157513.93	10.00	8036.40	673.10
inst_1000_60_00009	327002350.00	0.00	10.35	19.00	0.00	324423234.60	167043.18	10.02	3062.20	202.40	338560332.40	734725.66	10.00	7659.60	700.49
inst_1000_60_00010	268507556.40	5100.54	10.40	19.80	0.42	265680907.00	134435.98	10.01	3435.80	248.82	278863510.70	1050038.89	10.00	7851.70	354.08

Table 5: Results of Exchange Neighborhood

	Simulated Annealing					Variable Neighborhood Descent				
	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}	μ_{obj}	σ_{obj}	t	μ_{iter}	σ_{iter}
com_50_4_00001	77998.10	362.57	0.51	15940	302.40	76269.00	0.00	0.02	5.00	0.00
inst_50_4_00001	75283.63	58.95	0.51	16620	279.65	74247.00	0.00	0.02	5.00	0.00
inst_50_4_00002	24561.73	23.34	0.51	15600	0.00	23982.00	0.00	0.02	5.00	0.00
inst_50_4_00003	12577.40	25.31	0.51	15780	320.99	12278.00	0.00	0.02	5.00	0.00
inst_50_4_00004	7249.53	57.95	0.51	16160	219.09	6534.00	0.00	0.02	5.00	0.00
inst_50_4_00005	3915.93	15.90	0.51	15600	222.83	3580.00	0.00	0.02	5.00	0.00
inst_50_4_00006	3127.97	7.62	0.51	15560	152.22	2959.00	0.00	0.02	5.00	0.00
inst_50_4_00007	2446.60	74.86	0.51	15880	408.87	2044.50	4.25	0.03	5.00	0.00
inst_50_4_00008	1512.17	3.42	0.51	15740	375.64	1406.47	1.38	0.02	5.00	0.00
inst_50_4_00009	1486.67	5.06	0.51	15560	152.22	1307.00	8.63	0.02	5.00	0.00
inst_50_4_00010	1084.87	32.25	0.51	16000	454.86	852.93	1.57	0.02	5.00	0.00
com_200_20_00001	21773467.70	5131.66	2.93	29700	0.00	21458987.60	1410.65	1.72	4.87	0.35
inst_200_20_00001	22057139.23	11900.62	2.96	29700	0.00	21737583.60	130.74	1.66	4.93	0.25
inst_200_20_00002	7738077.80	1279.39	2.40	23760	4932.90	7607282.40	73.18	1.69	4.83	0.59
inst_200_20_00003	3889306.33	501.94	2.05	20130	1807.48	3827483.73	96.51	1.78	4.67	0.80
inst_200_20_00004	2257533.13	1198.97	2.03	19800	0.00	2196031.53	10.61	1.66	5.00	0.00
inst_200_20_00005	1394567.50	426.99	2.04	19800	0.00	1364813.33	33.95	1.58	4.97	0.18
inst_200_20_00006	1055410.30	1049.39	2.05	19800	0.00	999031.50	106.59	1.70	4.70	0.65
inst_200_20_00007	766819.23	531.86	2.04	19800	0.00	736962.00	23.05	2.07	3.10	1.32
inst_200_20_00008	577568.33	217.25	2.60	25740	4932.90	559793.30	76.40	1.54	5.00	0.00
inst_200_20_00009	453040.60	45.00	2.04	19800	0.00	436342.60	71.26	1.89	4.20	1.27
inst_200_20_00010	371252.60	103.09	2.07	19800	0.00	352006.40	36.21	1.81	4.80	0.61
com_500_40_00003	68440807.10	4307.19	14.79	62250	0.00	67520085.80	1124.72	5.02	2.00	0.00
com_500_40_00012	288772029.00	23594.84	14.95	62250	0.00	285345117.87	35129.71	5.01	2.00	0.00
com_500_40_00021	587489928.07	52857.33	14.92	62250	0.00	580094903.50	52847.32	5.01	2.00	0.00
inst_500_40_00001	37374110.50	3071.14	14.81	62250	0.00	36736413.80	4088.55	5.01	2.00	0.00
inst_500_40_00004	86509382.47	5563.92	14.89	62250	0.00	85241284.47	6286.40	5.02	2.00	0.00
inst_500_40_00007	155918365.10	11179.43	14.96	62250	0.00	153818923.50	6725.23	5.01	2.00	0.00
inst_500_40_00010	233468168.50	17927.83	15.14	62250	0.00	230791354.93	28285.47	5.01	2.00	0.00
inst_500_40_00013	317401610.30	17574.25	14.90	62250	0.00	314315272.70	11429.65	5.01	2.00	0.00
inst_500_40_00016	417162578.43	38640.09	14.93	62250	0.00	413009212.03	22083.82	5.01	2.00	0.00
inst_500_40_00019	512078245.97	89300.14	14.84	62250	0.00	507266128.23	6393.08	5.02	2.00	0.00
inst_1000_60_00001	14822971779.10	725738.98	129.09	249500	0.00	14731920241.40	206631.57	10.01	2.00	0.00
inst_1000_60_00002	5171204130.00	49716.78	125.95	249500	0.00	5133839046.30	297634.07	10.02	2.00	0.00
inst_1000_60_00003	2579810818.80	56844.12	132.09	249500	0.00	2557387921.00	114170.91	10.01	2.00	0.00
inst_1000_60_00004	1529951256.50	45587.86	129.71	249500	0.00	1515942003.50	47681.99	10.01	2.00	0.00
inst_1000_60_00005	1007370691.80	53995.58	132.18	249500	0.00	995762656.70	66528.68	10.01	2.00	0.00
inst_1000_60_00006	719018300.90	36365.91	131.56	249500	0.00	712278818.00	31515.31	10.02	2.00	0.00
inst_1000_60_00007	526144923.60	12499.38	130.92	249500	0.00	520144540.60	34205.49	10.02	2.00	0.00
inst_1000_60_00008	412915417.90	2728.92	129.89	249500	0.00	407210500.20	24189.69	10.02	2.00	0.00
inst_1000_60_00009	327373790.90	23354.51	130.22	249500	0.00	322906412.40	21810.46	10.03	2.00	0.00
inst_1000_60_00010	268840142.70	4932.31	129.92	249500	0.00	264965807.70	17010.18	10.01	2.00	0.00

Table 6: Results of Variable Neighborhood Search and Simulated Annealing

	Best Greedy Deterministic	Best Greedy Random k=10	Best GRASP k=10
com_50_4_00001	79610.00	80164.00	76270.00
inst_50_4_00001	75307.00	77356.00	74270.00
inst_50_4_00002	24569.00	26541.00	23982.00
inst_50_4_00003	12590.00	13102.00	12278.00
inst_50_4_00004	7284.00	7921.00	6583.00
inst_50_4_00005	3923.00	4322.00	3580.00
inst_50_4_00006	3131.00	3580.00	2959.00
inst_50_4_00007	2588.00	2766.00	2191.00
inst_50_4_00008	1513.00	1867.00	1435.00
inst_50_4_00009	1489.00	1743.00	1346.00
inst_50_4_00010	1227.00	1197.00	869.00
com_200_20_00001	21775940.00	21830856.00	21526705.00
inst_200_20_00001	22060987.00	22115589.00	21812939.00
inst_200_20_00002	7738678.00	7767250.00	7640151.00
inst_200_20_00003	3889565.00	3908558.00	3846782.00
inst_200_20_00004	2258014.00	2260342.00	2208916.00
inst_200_20_00005	1394712.00	1399824.00	1368909.00
inst_200_20_00006	1055928.00	1060554.00	1024394.00
inst_200_20_00007	766948.00	772835.00	750066.00
inst_200_20_00008	577661.00	582511.00	563412.00
inst_200_20_00009	453058.00	456413.00	442955.00
inst_200_20_00010	371300.00	372898.00	357823.00
com_500_40_00003	68443048.00	68493043.00	67850363.00
com_500_40_00012	288783726.00	288949130.00	286359338.00
com_500_40_00021	587513266.00	587631143.00	582847607.00
inst_500_40_00001	37374937.00	37408918.00	37037079.00
inst_500_40_00004	86510996.00	86555375.00	85668286.00
inst_500_40_00007	155924541.00	155985607.00	154492154.00
inst_500_40_00010	233475415.00	233622041.00	231588673.00
inst_500_40_00013	317406799.00	317607618.00	315344771.00
inst_500_40_00016	417182294.00	417290878.00	414269069.00
inst_500_40_00019	512123351.00	512243774.00	508398806.00
inst_1000_60_00001	14823503220.00	14825212418.00	14767540162.00
inst_1000_60_00002	5171232271.00	5172237038.00	5146610932.00
inst_1000_60_00003	2579838354.00	2580684086.00	2568699652.00
inst_1000_60_00004	1529974118.00	1530074258.00	1522756108.00
inst_1000_60_00005	1007405055.00	1007709987.00	1001334742.00
inst_1000_60_00006	719034385.00	719318010.00	715875203.00
inst_1000_60_00007	526150590.00	526370011.00	523455296.00
inst_1000_60_00008	412916980.00	413159085.00	410102511.00
inst_1000_60_00009	327382939.00	327539004.00	325306541.00
inst_1000_60_00010	268843223.00	268978638.00	266718650.00

Table 7: Best results for Greedy and GRASP

	Swap Neighborhood			Insert Neighborhood			Exchange Neighborhood		
	Best Improvement	First Improvement	Random Improvement	Best Improvement	First Improvement	Random Improvement	Best Improvement	First Improvement	Random Improvement
com_50_4_00001	76653.00	76653.00	85791.00	76269.00	76269.00	83937.00	76643.00	76637.00	84121.00
inst_50_4_00001	74906.00	74906.00	78638.00	74247.00	74247.00	81756.00	74931.00	74929.00	80129.00
inst_50_4_00002	24205.00	24205.00	27480.00	23982.00	23982.00	27572.00	24013.00	23982.00	27508.00
inst_50_4_00003	12315.00	12315.00	14447.00	12278.00	12278.00	14604.00	12350.00	12303.00	14301.00
inst_50_4_00004	6856.00	6856.00	8085.00	6534.00	6534.00	8204.00	6856.00	6753.00	7985.00
inst_50_4_00005	3584.00	3584.00	4689.00	3580.00	3580.00	4568.00	3594.00	3588.00	4788.00
inst_50_4_00006	3049.00	3046.00	3929.00	2959.00	2959.00	3960.00	2979.00	2963.00	3990.00
inst_50_4_00007	2530.00	2525.00	2775.00	2042.00	2042.00	2808.00	2441.00	2160.00	3357.00
inst_50_4_00008	1456.00	1452.00	2181.00	1407.00	1403.00	2245.00	1424.00	1421.00	2097.00
inst_50_4_00009	1433.00	1433.00	2014.00	1295.00	1295.00	2012.00	1382.00	1359.00	1907.00
inst_50_4_00010	1170.00	1170.00	1285.00	852.00	851.00	1315.00	884.00	855.00	1439.00
com_200_20_00001	21594503.00	21594484.00	22389515.00	21648533.00	21456936.00	22593643.00	21530406.00	21526793.00	22822937.00
inst_200_20_00001	21864884.00	21864896.00	22599861.00	21889705.00	21736170.00	22955536.00	21853052.00	21819521.00	22979656.00
inst_200_20_00002	7658818.00	7657447.00	7994215.00	7693198.00	7607193.00	8196245.00	7638944.00	7639092.00	8258918.00
inst_200_20_00003	3861866.00	3861779.00	4095989.00	3856826.00	3827280.00	4219858.00	3857004.00	3850807.00	4245296.00
inst_200_20_00004	2215603.00	2215250.00	2372205.00	2232247.00	2194223.00	2446391.00	2208470.00	2203232.00	2480660.00
inst_200_20_00005	1376106.00	1376300.00	1491778.00	1383700.00	1364784.00	1568983.00	1369639.00	1367739.00	1552344.00
inst_200_20_00006	1032744.00	1032556.00	1116736.00	1032317.00	998895.00	1152539.00	1021810.00	1016716.00	1155525.00
inst_200_20_00007	753009.00	752796.00	824455.00	754404.00	736926.00	868086.00	750032.00	745827.00	860567.00
inst_200_20_00008	565453.00	564521.00	638323.00	569067.00	559767.00	683662.00	561866.00	561862.00	696551.00
inst_200_20_00009	445665.00	445460.00	495687.00	446065.00	436223.00	525618.00	443270.00	441359.00	523901.00
inst_200_20_00010	360977.00	360712.00	410089.00	363519.00	351967.00	428928.00	358025.00	354254.00	426970.00
com_500_40_00003	67926405.00	67920367.00	68842956.00	68383595.00	67518423.00	73981624.00	68118401.00	67690079.00	72975551.00
com_500_40_00012	286597763.00	286592385.00	289604117.00	288530262.00	285308537.00	302990453.00	288004733.00	286079780.00	299843339.00
com_500_40_00021	583214450.00	583214145.00	588187033.00	587102726.00	579985350.00	603651291.00	586110562.00	581583818.00	600488573.00
inst_500_40_00001	37126642.00	37123438.00	37571444.00	37342570.00	36727371.00	40948040.00	37204034.00	36917677.00	40267868.00
inst_500_40_00004	85813227.00	85800431.00	86930049.00	86429110.00	85230519.00	93731868.00	86180854.00	85537203.00	91535573.00
inst_500_40_00007	154583375.00	154582169.00	156520085.00	155745644.00	153809935.00	164936676.00	155276701.00	154203080.00	162220585.00
inst_500_40_00010	231829044.00	231817648.00	234259144.00	233292869.00	230739480.00	246194041.00	232555289.00	231401447.00	243177303.00
inst_500_40_00013	315589596.00	315585432.00	318378057.00	317192068.00	314276248.00	332355576.00	316394934.00	314764020.00	328582864.00
inst_500_40_00016	414681252.00	414669169.00	418165002.00	416873152.00	412976727.00	433962626.00	416482137.00	414111536.00	430295113.00
inst_500_40_00019	509146887.00	509130379.00	513097736.00	511761046.00	507251984.00	529388872.00	510558226.00	507757776.00	525385487.00
inst_1000_60_00001	14771561793.00	14771527984.00	14826352706.00	14821539153.00	14731565908.00	15123715359.00	14816845989.00	14747957978.00	14937068046.00
inst_1000_60_00002	5148864915.00	5148844979.00	5173460411.00	5170513120.00	5133795677.00	5326879718.00	5168177323.00	5139656700.00	5223122883.00
inst_1000_60_00003	2570087708.00	2570127531.00	2581560343.00	2579273168.00	2557056990.00	2696631081.00	2578286384.00	2565063361.00	2621248847.00
inst_1000_60_00004	1523316432.00	1523315490.00	1530698046.00	1529687928.00	1516002128.00	1607785503.00	1528729192.00	1519219840.00	1555730623.00
inst_1000_60_00005	1001903829.00	1001883013.00	1008130482.00	1006952164.00	995647474.00	1065138727.00	1006307441.00	998121494.00	1029352620.00
inst_1000_60_00006	716228682.00	716236201.00	719642335.00	718879061.00	712304136.00	771373334.00	718271571.00	715000216.00	737130088.00
inst_1000_60_00007	523843481.00	523626402.00	526661822.00	526028126.00	520143317.00	565277744.00	525550588.00	522031535.00	540395442.00
inst_1000_60_00008	410410307.00	410471925.00	413282485.00	412765398.00	407191393.00	446838933.00	412490272.00	408613609.00	425234738.00
inst_1000_60_00009	325328030.00	325344635.00	327666845.00	327269610.00	322878366.00	355672860.00	327002350.00	324110011.00	337492714.00
inst_1000_60_00010	266861086.00	266861609.00	269110815.00	268687612.00	264929351.00	292819263.00	268505137.00	265512302.00	277191165.00

Table 8: Best results for Local Searches

	Simulated Annealing	Variable Neighborhood descent
com_50_4_00001	77164.00	76269.00
inst_50_4_00001	75009.00	74247.00
inst_50_4_00002	24473.00	23982.00
inst_50_4_00003	12512.00	12278.00
inst_50_4_00004	7096.00	6534.00
inst_50_4_00005	3861.00	3580.00
inst_50_4_00006	3104.00	2959.00
inst_50_4_00007	2297.00	2042.00
inst_50_4_00008	1495.00	1403.00
inst_50_4_00009	1470.00	1295.00
inst_50_4_00010	1018.00	851.00
com_200_20_00001	21752726.00	21456945.00
inst_200_20_00001	22000189.00	21737290.00
inst_200_20_00002	7733588.00	7607193.00
inst_200_20_00003	3887581.00	3827286.00
inst_200_20_00004	2253643.00	2196013.00
inst_200_20_00005	1392564.00	1364769.00
inst_200_20_00006	1051857.00	998899.00
inst_200_20_00007	764059.00	736921.00
inst_200_20_00008	576795.00	559401.00
inst_200_20_00009	452863.00	436229.00
inst_200_20_00010	370887.00	351967.00
com_500_40_00003	68424343.00	67518172.00
com_500_40_00012	288696650.00	285303015.00
com_500_40_00021	587232719.00	580028186.00
inst_500_40_00001	37358318.00	36728514.00
inst_500_40_00004	86482441.00	85228180.00
inst_500_40_00007	155890408.00	153810069.00
inst_500_40_00010	233391667.00	230740327.00
inst_500_40_00013	317313288.00	314295786.00
inst_500_40_00016	417040899.00	412976344.00
inst_500_40_00019	511816735.00	507250602.00
inst_1000_60_00001	14821553894.00	14731642278.00
inst_1000_60_00002	5171082854.00	5133330608.00
inst_1000_60_00003	2579660896.00	2557206317.00
inst_1000_60_00004	1529825582.00	1515877374.00
inst_1000_60_00005	1007279760.00	995638503.00
inst_1000_60_00006	718918253.00	712235364.00
inst_1000_60_00007	526110890.00	520097613.00
inst_1000_60_00008	412908650.00	407169538.00
inst_1000_60_00009	327308030.00	322881574.00
inst_1000_60_00010	268828789.00	264937903.00

Table 9: Best results for Simulated Annealing and Variable Neighborhood Descent